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## COMPACTIFICATION AND COMPACTOIDIFICATION

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**Abstract.** After discussing some of the many ways to get the Banaschewski compactification  $\beta_0 T$  of an arbitrary ultraregular space  $T$ , we develop another construction of  $\beta_0 T$  in Th. 2.1. Using those ideas, we develop an analog of  $\beta_0 T$ —what we call a *compactoidification*  $\kappa T$  of an ultraregular space  $T$  in Sec. 3;  $\kappa T$  is, in essence, a complete absolutely convex compactoid ‘superset’ of  $T$  to which continuous maps of  $T$  with precompact range into any complete absolutely convex compactoid subset may be ‘continuously extended.’

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### 1 The Many Faces

For any topological spaces  $X$  and  $Y$ ,  $C(X, Y)$  and  $C^*(X, Y)$  denote the spaces of continuous maps of  $X$  into  $Y$  and the continuous maps of  $X$  into  $Y$  with relatively compact range, respectively. To say that a topological space  $X$  is *ultraregular* or *ultranormal* means, respectively, that the clopen sets are a basis or disjoint closed subsets of  $X$  may be separated by clopen sets. A synonym for *ultraregular* is *0-dimensional*. We have a slight preference for the former in order to avoid confusion with other notions of dimension. Throughout the discussion,  $T$  denotes at least a Hausdorff space. For an ultraregular space  $E$  containing at least two points and ultraregular  $T$ , B. Banaschewski [2] discovered a compactification  $\beta_0 T$  of  $T$  in which every  $x \in C^*(T, E)$  may be continuously extended to  $\beta_0 x \in C(\beta_0 T, E)$ .  $\beta_0 T$  is nowadays usually called the *Banaschewski compactification* of  $T$ . It functions as the natural analog of the Stone-Čech compactification ( $\beta_0 T$  is  $\beta T$  for ultranormal  $T$ ) in non-Archimedean analysis. Like the Stone-Čech compactification, the Banaschewski compactification is a protean entity, assuming many different guises. We discuss some of them in this section and then develop a new one in Sec. 2.

#### 1.1 As a completion

Let  $E$  be an ultraregular space containing at least two points and let  $T$  be ultraregular. Let  $C^*(T, E)$  denote the weakest uniform structure on  $T$  making each  $x \in C^*(T, E)$  uniformly continuous into the compact space  $\text{cl } x(T)$  equipped with its unique compatible uniform

structure. By [1], pp. 92-93, since  $T$  is ultraregular,  $C^*(T, E)$  is compatible with the topology on  $T$  and  $C^*(T, E)$  is a precompact uniform structure on  $T$ . Since  $C^*(T, E)$  is precompact, its completion  $\beta_0 T$  is compact and is called the *Banaschewski compactification* of  $T$ .  $\beta_0 T$  is ultranormal ([2], p. 131, Satz 2 or [1], p. 93, Theorem 1)—hence ultraregular—and, by the usual process of extension by continuity from a dense subspace to the whole space, each  $x \in C^*(T, E)$  may be continuously extended to a unique continuous function  $\beta_0 x \in C^*(\beta_0 T, E)$ .  $\beta_0 T$  is unique in a sense we discuss in the context of *E-compactifications* (Th. 1.6). At this point the reader may find the notation  $\beta_0 T$  curious. Why  $\beta_0 T$  and not  $\beta_E T$ ? As long as  $E$  is ultraregular and contains at least two points ([1], p. 93, [8], pp. 240-243), the uniformity  $C^*(T, E)$  does not depend on  $E$ ! A fundamental system of entourages for  $C^*(T, E)$ , no matter what  $E$  is, is defined by the sets

$$V_{\mathcal{P}} = \bigcup \{V \times V : V \in \mathcal{P}\}$$

where  $\mathcal{P}$  is any finite open (therefore clopen) cover of  $T$  by pairwise disjoint sets. The completion of  $T$  with respect to this uniformity is the way Banaschewski obtained  $\beta_0 T$ . The definition of  $\beta_0 T$  as the completion of  $C^*(T, E)$  where  $E$  is the discrete space of integers was first given in [7], though the idea of treating compactifications as completions is due to Nachbin. The connection with the Stone-Čech compactification is the following.

**Definition 1.1** *Let  $\mathcal{P}$  be a finite clopen cover of a topological space  $S$  by pairwise disjoint sets and let  $V$  denote the uniformity generated by  $V_{\mathcal{P}}$ . We say that  $S$  is strongly ultraregular if  $V = C^*(S, \mathbb{R})$ .*

**Theorem 1.2** ([8], pp. 251-2) (a) *Every ultranormal  $T_1$ -space  $S$  is strongly ultraregular.*  
 (b) *If a topological space  $S$  is strongly ultraregular then  $\beta_0 S = \beta S$ .*

## 1.2 As an E-Compactification

Tihonov proved that a completely regular space  $T$  may be characterized as one that is homeomorphic to a subspace of a product  $[0, 1]^m$  of unit intervals. Even though his name is not associated with it, he created the first version of the Stone-Čech compactification  $\beta T$  of  $T$  by then taking the closure of  $T$  in  $[0, 1]^m$ . Engelking and Mrówka [5] developed analogous notions of *E-completely regular space*  $T$  and *E-compactification*  $\beta_E T$ . Let  $S$  and  $E$  be two topological spaces.  $S$  is called *E-completely regular* if it is homeomorphic to a subspace of the  $m$ -fold topological product  $E^m$  for some cardinal  $m$ . If  $E = \mathbb{R}$  or  $[0, 1]$ , this is the familiar notion of complete regularity. With  $\mathbf{2}$  denoting the discrete space  $\{0, 1\}$ , it happens that

**Theorem 1.3** ([16], p. 17) *A topological space  $S$  is 2-completely regular if and only if it is an ultraregular  $T_0$ -space.*

An *E-compact* space is one which is homeomorphic to a closed subspace of a topological product  $E^m$  for some cardinal  $m$ . The 2-compact spaces are characterized as follows:

**Theorem 1.4** ([5], p. 430, Example (iii)) *A topological space  $S$  is 2-compact if and only if it is compact and ultraregular.*

An  $E$ -compactification  $\beta_E T$  of an  $E$ -completely regular space  $T$  is

- (1) an  $E$ -compact space which contains  $T$  as a dense subset and
- (2) ('the  $E$ -extension property') each  $x \in C(T, E)$  may be extended to  $\beta_E x \in C(\beta_E T, E)$ .

The following analogs of properties of the Stone-Čech compactification obtain for  $E$ -compactifications.

**Theorem 1.5** ([5], p. 433, Theorem 4, [16], pp. 25-27, 4.3 and 4.4). *An  $E$ -completely regular (Hausdorff) space  $T$  has a Hausdorff  $E$ -compactification  $\beta_E T$  with the following properties:*

- (a) *If  $S$  is an  $E$ -compact space then every continuous function  $x : T \rightarrow S$  has a continuous extension  $\bar{x} : \beta_E T \rightarrow S$ .*
- (b) *The space  $\beta_E T$  is unique in the sense that if  $S$  is an  $E$ -compact space containing  $T$  as a dense subset and such that every continuous  $x : T \rightarrow E$  has a continuous extension to  $S$ , then  $S$  is homeomorphic to  $\beta_E T$  under a homeomorphism that is the identity on  $T$ .*
- (c)  *$T$  is  $E$ -compact if and only if  $T = \beta_E T$ .*

How does this apply to  $\beta_0 T$ ? Ultraregular spaces  $T$  are 2-completely regular by Th. 1.3. Since  $\beta_0 T$  is compact and ultranormal, it follows that  $\beta_0 T$  is 2-compact by Th. 1.4. Therefore, by Th. 1.5(b) it follows that

**Theorem 1.6** UNIQUENESS OF  $\beta_0 T$ .  *$\beta_0 T$  is homeomorphic to  $\beta_2 T$  under a homeomorphism that is the identity on  $T$ , as would be any ultraregular compactification of an ultraregular  $T$  with the  $E$ -extension property.*

### 1.3 As a Space of Characters

Let  $F$  be an ultraregular Hausdorff topological field so that  $X = C^*(T, F)$  may be considered as an  $F$ -algebra. A *character* of  $X$  is a nonzero algebra homomorphism from  $X$  into  $F$ . Let the set  $H$  of characters of  $X$  be equipped with the weakest topology for which the maps  $H \rightarrow F$ ,  $h \mapsto h(x)$ , are continuous for each  $x \in C^*(T, F)$ . For each  $p \in \beta_0 T$ , let  $p^\wedge$  denote the *evaluation map* at  $p$ , the map  $C^*(T, F) \rightarrow F$ ,  $x \mapsto \beta_0 x(p)$ . It is trivial to verify that each  $p^\wedge$  is a character of  $C^*(T, F)$ . But more is true: You get all the characters of  $C^*(T, F)$  this way. In fact, the map

$$\begin{aligned} A : \beta_0 T &\longrightarrow H \\ p &\longmapsto p^\wedge \end{aligned}$$

establishes a homeomorphism between  $\beta_0 T$  and  $H$ . The details may be found in [1], Theorem 3 and [8], Theorem 8.15.



## 1.4 Characters Again

Once again  $\beta_0 T$  is realized as a space of nonzero homomorphisms—ring homomorphisms this time—into the very simple (discrete) field  $\mathbf{2}$  with 2 elements.

A commutative ring  $X$  with identity in which each element is idempotent is called a *Boolean ring*. A subcollection  $\mathcal{X}$  of the set of subsets of a given set  $T$  which is closed under union, intersection and set difference of any two of its members is called a *ring of sets*. Such a collection forms a ring in the usual algebraic sense if addition and multiplication are taken to be symmetric difference and intersection, respectively. If the sets in  $\mathcal{X}$  cover  $T$  then  $\mathcal{X}$  is called a *covering ring*. Since  $\mathcal{X}$  must have a multiplicative identity (i.e., with respect to intersection) any covering ring must contain  $T$  as an element. Any covering ring  $\mathcal{X}$  generates (in the sense that it is a subbase for) a ultraregular topology on  $T$ ; the topology is ultraregular since the complement  $T - A$  of any open set (member of  $\mathcal{X}$ ) must belong to  $\mathcal{X}$ . In the converse direction, the class  $\text{Cl}(T)$  of clopen subsets obviously constitutes a covering ring of any topological space  $T$ .

Let  $X$  be a Boolean ring and endow  $2^X$  with the product topology. The *Stone space*  $S(X)$  of the Boolean ring  $X$  is the subspace of  $2^X$  of all nonzero ring homomorphisms of  $X$  into  $\mathbf{2}$ .  $S(X)$  is called the Stone space because of Stone's use of it in his remarkable characterization of compact ultraregular spaces.

THE STONE REPRESENTATION THEOREM ([12], Theorem 4, [12], [4] p.227 or [6], pp. 77-80) If  $T$  is a compact ultraregular space, then  $T$  is homeomorphic to the Stone space of the Boolean ring  $\text{Cl}(T)$  of clopen subsets of  $T$ . Conversely, the Stone space  $S(X)$  of any Boolean ring  $X$  is a compact ultraregular Hausdorff space and  $X$  is ring-isomorphic to the Boolean ring  $\text{Cl}(T)$  of clopen subsets of  $S(X)$ .

If  $T$  is ultraregular then  $\beta_0 T$  is the Stone space of  $\text{Cl}(T)$ . Indeed, the map  $\beta : T \rightarrow S(\text{Cl}(T))$ ,  $t \mapsto \beta t$ , defined for  $t \in T$  and  $K \in \text{Cl}(T)$  by

$$(\beta t)(K) = \begin{cases} 1 \in \mathbf{2} & t \in K \\ 0 \in \mathbf{2} & t \notin K \end{cases}$$

is a homeomorphism of  $T$  onto a dense subset of the compact ultraregular Hausdorff space  $S(\text{Cl}(T))$ .

## 1.5 As a Space of Measures

Let  $T$  be ultraregular and let  $\text{Cl}(T)$  be the ring (algebra, actually, since  $T \in \text{Cl}(T)$ ) of clopen subsets of  $T$ , and let  $F$  be an ultraregular Hausdorff topological field. A *0-1 measure on  $T$*  is a finitely additive set function  $m : \text{Cl}(T) \rightarrow \{0, 1\} \subset F$  satisfying the condition:

$$m(U) = 0 \quad \text{and} \quad U \supset V \in \text{Cl}(T) \implies m(V) = 0$$

in other words, that clopen subsets of sets of measure 0 also have measure 0. Measures  $m_t$  'concentrated at points  $t \in T$ ' (also called 'purely atomic' or 'the point mass at  $t$ ') which

are 1 on a clopen set  $U$  if  $t \in U$  and 0 otherwise are 0-1 measures on  $T$ . The *weak clopen topology* for the collection  $M$  of all 0-1 measures on  $T$  has as a neighborhood base  $m_0 \in M$  sets of the form

$$V(m_0; S_1, \dots, S_n) = \{m \in M : m(S_j) = m_0(S_j), j = 1, \dots, n\}$$

where the  $S_j$  are clopen sets and  $n \in \mathbb{N}$ . It is trivial to verify that the map  $t \rightarrow m_t$  is a homeomorphism of  $T$  into  $M$ . Using the techniques of [1] one can demonstrate that  $M$  is a compact ultranormal Hausdorff space to which any  $x \in C^*(T, F)$  may be continuously extended. It follows that  $\beta_0 T = M$  in the sense of Th. 1.6.

Last, let us mention that  $\beta_0 T$  may also be realized as a Wallman compactification utilizing the lattice of clopen subsets of  $T$ .

## 2 A New Approach

A construction of  $\beta_0 T$  using the methods of non-Archimedean functional analysis is presented in Theorem 2.1. The proof hinges on the fact that, for a local field  $F$ , if  $U$  is a neighborhood of 0 in a locally  $F$ -convex space  $X$  then its polar  $U^\circ$  is  $\sigma(X', X)$ -compact ([15], Th. 4.11). Note that  $\sigma(X', X)$  is ultraregular since the seminorms  $p_x(f) = |f(x)|$ ,  $x \in X$ ,  $f \in X'$ , are non-Archimedean.

**Theorem 2.1** *Let  $F$  be a local field, let  $T$  be ultraregular and let  $C^*(T, F)$  denote the sup-normed space of all continuous  $F$ -valued functions on  $T$  with relatively compact range. There is an ultranormal compactification  $\beta_0 T$  of  $T$  such that any  $x \in C^*(T, F)$  may be continuously extended to a function  $\beta_0 x \in C(\beta_0 T, F)$ .*

**Proof.** For  $t \in T$ , let  $t^\wedge$  denote the evaluation map  $x \mapsto x(t)$  for any  $x \in C^*(T, F)$ . We note that each such  $t^\wedge$  is a continuous linear form (algebra homomorphism, actually) and is of norm one. Thus  $T^\wedge = \{t^\wedge : t \in T\} \subset U$  where  $U$  denotes the unit ball of the norm-dual  $C^*(T, F)'$  of  $C^*(T, F)$ . Furthermore, the map  $i : T \rightarrow C^*(T, F)'$ ,  $t \mapsto t^\wedge$ , embeds  $T$  homeomorphically in  $C^*(T, F)'$  endowed with its weak-\* topology by the following argument. The map  $i$  is obviously injective. If a net  $t_s \rightarrow t \in T$  then  $x(t_s) \rightarrow x(t)$  for any  $x \in C^*(T, F)$ ; hence  $t_s^\wedge \rightarrow t^\wedge$  and therefore  $i$  is continuous. To see that  $i$  is a homeomorphism onto  $i(K)$ , let  $K$  be a closed subset of  $T$ . Since  $T$  is ultraregular, if  $t \notin K$  then there exists  $x \in C^*(T, F)$  such that  $x(t) = 0$  and  $|x(K)| = r > 1$ . Hence the polar  $\{x\}^\circ$  of  $\{x\}$  is a neighborhood of  $t^\wedge$  disjoint from  $K^\wedge$  and  $K^\wedge$  is a closed subset of  $i(K)$ . As  $U$  is the polar of the unit ball of  $C^*(T, F)$ , it follows that  $U$  is weak-\*compact ([15], Th. 4.11). Therefore the closure  $cT$  in  $U$  of (the homeomorphic image of)  $T^\wedge$  is compact in  $C^*(T, F)'$  endowed with the weak-\* topology. As to the continuous extendibility of  $x \in C^*(T, F)$ , consider the canonical image  $Jx$  of  $x$  in the second algebraic dual of  $C^*(T, F)$ , i.e., for any  $f \in C^*(T, F)'$ ,  $Jx(f) = f(x)$ . Clearly  $Jx$  is weak-\*continuous on  $C^*(T, F)'$ ; so, therefore, is its restriction  $\beta_0 x = Jx|_{cT}$ . Should this be called  $c_F T$  rather than  $cT$ ? No topologically significant changes occur for different  $F$ 's: the compactness of the ultraregular space  $cT$  and the fact that  $T$  is  $C^*$ -embedded in  $cT$  imply that  $cT = \beta_0 T$  by Th. 1.6.

### 3 Compactoidification

In this section we construct a *compactoidification*  $\kappa T$  of an ultraregular space  $T$ .  $(F, |\cdot|)$  denotes a complete nontrivially ultravalued field throughout. As usual, we abbreviate ' $F$ -convex' to 'convex.' A map  $f$  defined on an absolutely convex subset  $A$  of a vector space over  $F$  with values in some absolutely convex set in a vector space over  $F$  is called *affine* if  $f(ax + by) = af(x) + bf(y)$  for all  $x, y \in A$  and all  $a, b \in F$  with  $|a| \leq 1$  and  $|b| \leq 1$ .

**Definition 3.1** A compactoidification of an ultraregular space  $T$  is a pair  $(i, \kappa T)$  where  $\kappa T$  is a complete absolutely convex compactoid subset of some Hausdorff locally convex space  $E$  over  $F$  and  $i : T \rightarrow \kappa T$  is a continuous map with precompact range for which following extendibility property holds: For any complete absolutely convex compactoid subset  $A$  of some Hausdorff locally convex space  $E$  over  $F$  and any continuous map  $j : T \rightarrow A$  with precompact range, there exists a unique continuous affine map  $J : \kappa T \rightarrow A$  such that  $J \circ i = j$ .

$$\begin{array}{ccc} & \kappa T & \\ i \uparrow & \searrow J & \\ T & \xrightarrow{j} & A \end{array}$$

**Theorem 3.2** A compactoidification is unique in the following natural sense: if  $(i_1, \kappa_1 T)$  and  $(i_2, \kappa_2 T)$  are compactoidifications of  $T$  then there exists a unique affine homeomorphism  $J_1 : \kappa_1 T \rightarrow \kappa_2 T$  such that  $J_1 \circ i_1 = i_2$ . Moreover, the map  $i$  must be injective.

**Proof.** By definition, there exist unique continuous affine maps  $J_1$  and  $J_2$  such that  $J_2 \circ i_1 = i_2$  and  $J_1 \circ i_2 = i_1$ . Thus,  $J_1 \circ (J_2 \circ i_1) = J_1 \circ i_2 = i_1$ .

$$\begin{array}{ccc} & \kappa_1 T & \\ i_1 \uparrow & \searrow J_2 & \\ T & \xrightarrow{i_2} & \kappa_2 T \end{array}$$

Since the identity map  $I_1 : t \mapsto t$  of  $\kappa_1 T$  onto  $\kappa_1 T$  also satisfies  $I_1 \circ i_1 = i_1$ , it follows from the uniqueness that  $I_1 = J_1 \circ J_2$ . Similarly,  $I_2 = J_2 \circ J_1$  where  $I_2$  is the identity map of  $\kappa_2 T$  onto  $\kappa_2 T$ . It follows that  $J_1$  is a homeomorphism of  $\kappa_1 T$  onto  $\kappa_2 T$  and  $J_2$  is its inverse. If  $i_1(t_1) = i_1(t_2)$  then  $i_2(t_1) = J_1 \circ i_1(t_1) = J_1 \circ i_1(t_2) = i_2(t_2)$  so if one of the maps  $i$  is 1-1, all such  $i$  must be. As shown in Theorem 3.3, there is an  $i$  that is 1-1.

In the notation of Sec. 2:

**Theorem 3.3** Let  $T$  be ultraregular and let the continuous dual  $C^*(T, F)'$  of  $C^*(T, F)$  carry the weak-\* topology. Then

- (a) the closed absolutely convex hull  $\kappa T$  of  $T^\wedge$  is the unit ball  $U$  of  $C^*(T, F)'$  and
- (b) the pair  $(i, \kappa T)$  is a compactoidification of  $T$ .

**Proof.** Clearly the absolute convex hull  $B$  of  $T^\wedge$  is contained in the unit ball  $U$  of  $C^*(T, F)'$ . Since  $U$  is a complete compactoid by the  $p$ -adic Alaoglu theorem ([9], Prop.



3.1), so, therefore, is the closed absolutely convex hull  $\kappa T$  of the compact set  $\text{cl } T^\wedge$ . It follows from [10], Prop. 1.3 that  $B$  is edged (i.e., if the valuation of  $F$  is dense then  $\text{cl } B = \bigcap \{a(\text{cl } B) : a \in F, |a| > 1\}$ ) and therefore ([9], Th. 4.7) a polar set in  $C^*(T, F)'$ . If  $\text{cl } B \neq U$  there must exist  $g \in C^*(T, F)''$  such that  $|g| \leq 1$  on  $B$  and  $|g(f)| > 1$  for some  $f \in U - \text{cl } B$ . Since  $g$  must be an evaluation map determined by some point  $x \in C^*(T, F)$  by [9], Lemma 7.1, we have found an  $x$  such that  $|x(t)| = |t^\wedge(x)| \leq 1$  for all  $t \in T$  but  $|f(x)| > 1$ . As this contradicts  $\|f\| \leq 1$ , the proof of (a) is complete.

(b) As in the proof of Th. 2.1,  $i$  is a homeomorphism onto the precompact set  $T^\wedge$ . To verify the extendibility requirement, let  $A$  be a complete absolutely convex compactoid and let  $j : T \rightarrow A$  be continuous with precompact range. We define the affine extension  $J$  of  $j$  on the absolutely convex hull  $B$  of  $T^\wedge$  by taking  $J(\sum_{i=1}^n a_i t_i^\wedge) = \sum_{i=1}^n a_i j(t_i)$  for  $a_i \in F, |a_i| \leq 1, i = 1, \dots, n$ . The definition makes sense because the  $t_i^\wedge$  are linearly independent for distinct  $t_i$ . Evidently  $j = J \circ i$ . To prove the continuity of  $J$ , let  $s \rightarrow \mu_s = \sum_{i=1}^n a_i^s t_i^s$  be a net in  $B$  convergent to 0 in the weak-\* topology. Let  $[A]$  denote the linear span of  $A$  and note that for any  $f \in [A]'$ , the map  $f \circ j \in C^*(T, F)$ , since  $j(T)$  is precompact. Thus,

$$f(J(\mu_s)) = f\left(\sum_{i=1}^n a_i^s j(t_i^s)\right) = \sum_{i=1}^n a_i^s f(j(t_i^s)) = \mu_s(f \circ j) \rightarrow 0$$

and we conclude that  $J(\mu_s) \rightarrow 0$  in the weak topology of  $[A]$ . As  $A$  is of countable type, hence a polar space, the weak topology coincides with the initial one on the compactoid  $A$  ([9], Th. 5.12) so  $J(\mu_s) \rightarrow 0$  in  $A$ . By continuity and 'affinity,'  $J$  extends uniquely to a continuous affine map of  $\text{cl } B = \kappa T$  into  $A$ , since  $A$  is complete.

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